# The Construction Schemes for Fault-Tolerance Directed Hamiltonian Graphs

Chun-Nan Hung and Guan-Yu Lai

Department of Computer Science and Information Engineering Da-Yet University Changhua, Taiwan, R.O.C

**Abstract**

A directed graph is Hamiltonian if it contains a directed Hamiltonian cycle. A directed graph $G = (V, E)$ is $k$-arc-fault-tolerant Hamiltonian if for every $F \subseteq E$ with $|F| \leq k$, $G-F$ is Hamiltonian. A directed graph $G = (V, E)$ is $k$-fault-tolerant Hamiltonian if for every $F \subseteq (E \cup V)$ with $|F| \leq k$, $G - F$ is Hamiltonian. In this paper, we will propose some construction schemes for fault-tolerant Hamiltonian directed graphs.

Let $G = (V, E)$ be a directed $r$-regular graph and $v \in V$ be an arbitrary vertex. Let $X = \{x_1, x_2, \cdots, x_r\}$ be the set of vertices adjacent to $v$ and $Y = \{y_1, y_2, \cdots, y_r\}$ be the set of vertices adjacent from $v$. The arc set $F \subset E$ is the set of faulty arcs with $|F| = f$ for $f \leq r - 1$. The directed graph $G$ is **joinable** if for every vertex $v \in V$ there exists a subset $X_a \subset X$ with $|X_a| = r - f$, for every vertex $x_j \in X_a$ there exist $r - f$ vertices $y_{j_1}, ..., y_{j_{r-f}}$ of $Y$ such that every pair of arcs $\langle x_j, v\rangle$ and $\langle v, y_{j_i}\rangle$ can be passing through by some Hamiltonian cycle in $G-F$ for $1 \leq i \leq r-f$. Thus, a $r$-regular joinable digraph is $(r-1)$-arc-fault-tolerance Hamiltonian. Let $G = (V, E)$ and $H = (U, A)$ be two directed $r$-regular graph and $v \in V, u \in U$ be arbitrary vertices. Let $X = \{x_1, x_2, ..., x_r\}$ and $W = \{w_1, w_2, ..., w_r\}$ be the sets of vertices adjacent to $v$ and $u$, respectively. Let $Y = \{y_1, y_2, ..., y_r\}$ and $Z = \{z_1, z_2, ..., z_r\}$ be the sets of vertices adjacent from $v$ and $u$, respectively. The **vertex-join** of $G$ and $H$ on the vertices on $v$ and $u$ is $K = (T, D)$ such that the vertex set $T = V \cup U - \{v, u\}$ and the arc set $D = E \cup A \cup \{\langle x_i, z_i\rangle, \langle w_i, y_i\rangle |$ for $1 \leq i \leq r\} - \{\langle x_i, v\rangle, \langle v, y_i\rangle, \langle w_i, u\rangle, \langle u, z_i\rangle |$ for $1 \leq i \leq r\}$. In this paper, we will show that the vertex-join of two $r$-regular joinable directed graphs is also $r$-regular joinable. Thus, the vertex-join is the important construction scheme for arc-fault-tolerance Hamiltonian digraphs. Furthermore, we will also show that the vertex-join is the construction scheme for fault-tolerance Hamiltonian digraphs.

**Keywords:** directed Hamiltonian graphs, arc-fault-tolerance, fault-tolerance, joinable, vertex-join.

E-mail address: spring@mail.dyu.edu.tw and R1006002@cloud.dyu.edu.tw