Recent progress on spectral graph theory

Bo Ning

College of Computer Science, Nankai University

Abstract

In this talk, we will survey our recent progress on some open problems from spectral graph theory. We first consider the problem of determining tight spectral radius conditions for a cycle of length ℓ in a graph of order n for each $\ell \in [3, n]$. We determine tight spectral radius conditions for C_ℓ where ℓ belongs to an interval of the form $[n - \Theta(\sqrt{n}), n]$. As a main tool, we prove a stability result of a theorem due to Woodall, which states that for a graph G of order $n \geq 2k + 3$ where $k \geq 0$ is an integer, if $e(G) > \binom{n-k-1}{2} + \binom{k+2}{2}$ then G contains a C_ℓ for each $\ell \in [3, n - k]$. Secondly, we will present some quantitative counting results for previous spectral theorems on triangles and quadrilaterals due to Nosal and Nikiforov, respectively. The last part is devoted to an open problem of Nikiforov (2008): What is the maximum C such that for all positive $\varepsilon < C$ and sufficiently large n, every graph G of order n with spectral radius $\rho(G) > \sqrt{\lfloor \frac{n^2}{4} \rfloor}$ contains a cycle of length ℓ for each integer $\ell \in [3, (C - \varepsilon)n]$. We will mention our recent progress on this problem. (Joint work with Binlong Li and Mingqing Zhai)

E-mail address: bo.ning@nankai.edu.cn