

# New truncated theorems for three classical theta function identities

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## Abstract

In 2012, Andrews and Merca derived a truncated version of Euler's pentagonal number theorem. Their work inspired several mathematicians to work on truncated theta series including Guo and Zeng, who examined two other classical theta series identities of Gauss. In this paper, revisiting these three theta series identities of Euler and Gauss, we obtain new truncated theorems. As corollaries of our results, we obtain infinite families of linear inequalities involving the partition function, the overpartition function and the pod function. These inequalities yield the positive result of Andrews and Merca on the partition function as well as a conjecture on the overpartition function, which was posed by Andrews–Merca and Guo–Zeng, and proved independently by Mao and Yee. We will also provide a unified combinatorial treatment for our results. (Join work with A.J. Yee and Xiang Zhao)

**Keywords:** theta functions, partitions, truncated sums.