Connected graphs of fixed order and size with maximal A_α-index: The one-dominating-vertex case

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Abstract

For any real number $\alpha \in [0, 1]$, by the A_α-*matrix* of a graph G we mean the matrix $A_\alpha(G) = \alpha D(G) + (1 - \alpha)A(G)$, where $A(G)$ and $D(G)$ denote respectively the adjacency matrix and the diagonal matrix of vertex degrees of G. The largest eigenvalue of $A_\alpha(G)$ is called the A_α-index of G. Chang and Tam (2011) have proved that for every pair of integers n, k with $-1 \leq k \leq n - 3$, $H_{n,k}$, the graph obtained from the star $K_{1,n-1}$ by joining a vertex of degree 1 to $k+1$ other vertices of degree 1, is the unique connected graph that maximizes the Q-index (i.e., the signless Laplacian spectral radius or, equivalently, the $A_{\frac{1}{2}}$-index) over all connected graphs with n vertices and $n + k$ edges. In this paper it is proved that for every pair of integers n, k with $-1 \leq k \leq n - 3$, when $\frac{1}{2} < \alpha < 1$ or $\alpha = \frac{1}{2}$ and $k \neq 2$, the graph $H_{n,k}$ is the unique connected graph that maximizes the A_α-index over all connected graphs with n vertices and $n + k$ edges. This work extends (and also provides an alternative proof for) the above-mentioned result of Chang and Tam. A complete overview of the history of the maximal index problems is also given.

Keywords: Maximal A_α-index problem; Maximal graph; Neighborhood equivalence class

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