Strict Log-Subadditivity for Overpartition Rank

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Abstract

Bessenrodt and Ono initially found the strict log-subadditivity of partition function $p(n)$, that is, $p(a + b) < p(a)p(b)$ for $a, b > 1$ and $a + b > 9$. Many other important partition statistics are proved to enjoy similar properties. Lovejoy introduced the overpartition rank as an analog of Dyson ̧es rank for partitions from the q-series perspective. Let $\overline{N}(a, c, n)$ denote the number of overpartitions with rank congruent to a modulo c. Ciolan computed the asymptotic formula of $\overline{N}(a, c, n)$ and showed that $\overline{N}(a, c, n) > \overline{N}(b, c, n)$ for $0 \leq a < b \leq \lfloor \frac{c}{2} \rfloor$ and n large enough if $c \geq 7$. In this talk, we derive an upper bound and a lower bound of $\overline{N}(a, c, n)$ for each $c \geq 3$ by using the asymptotics due to Ciolan. Consequently, we establish the strict log-subadditivity of $\overline{N}(a, c, n)$ analogous to the partition function $p(n)$. This work is joint with Ying Zhong.

Keywords: Rank, Overpartition, Strict Log-Subadditivity

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