Saturation of high dimensional 0-1 matrices

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Abstract

Given a 0-1 matrix P, another 0-1 matrix M is *P-saturated* if M does not contain a submatrix that can be turned into P by flipping any number of its 1-entries to 0-entries, and flipping any 0-entry of M to 1-entry creates a copy of P. Matrix M is *P-semisaturated* if flipping any 0-entry of M to 1-entry creates a new copy of P. The function $(s)sat(n, P)$ is the minimum possible number of 1-entries in a $n \times n$ 0-1 matrix that is P-(semi)saturated.

We will go over existing results on these functions for two-dimensional matrices, e.g., Fulek and Keszegh showed that $sat(n, P)$ is either $O(1)$ or $\Theta(n)$. Then we will introduce our works that generalize them to multidimensional matrices. In particular, up to a constant multiplicative factor we fully settle the problem of characterizing the semisaturation function of families of d-dimensional 0-1 matrices, which we prove to always be $\Theta(n^r)$ for some integer $r \in [0, d-1]$.

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