Some determinantal representations of generating functions

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Abstract

Munarini recently showed that the derangement polynomial $d_n(q) = \sum_{\sigma \in \mathcal{D}_n} q^{\mathrm{maj}(\sigma)}$ is expressible as the determinant of either an $n \times n$ tridiagonal matrix or an $n \times n$ lower Hessenberg matrix. Qi, Wang and Guo showed that the classical derangement number $d_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$ is expressible as a tridiagonal determinant of order $n+1$.

By applying the approach of Munarini, the original and q-extended approach of Qi-Wang-Guo, we show in this talk determinantal expressions for various generating functions of permutations, signed or not, including Eulerian polynomials, derangement polynomials as well as derangement numbers.

Keywords: Derangement polynomial, Eulerian polynomial, determinant, tridiagonal, lower Hessenberg.

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