On bipartite graphs with the minimum number of spanning trees

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Abstract

In this report, we investigate the problem of characterizing the graphs have the minimum number of spanning trees among all simple undirected connected bipartite graphs with fixed cyclomatic number. Denote by $K^c_{a;b}$ the graph obtained from the complete bipartite graph $K_{a,b+1}$ by deleting $a-c$ edges all connected to the same vertex with degree a, and by $\mathbb{S}(G)$ the skeleton of the graph G, the maximum induced subgraph of G whose every vertex contains no pendent trees. We show that the skeleton of each graph with the minimum number of spanning trees among all simple undirected connected bipartite graphs with cyclomatic number m is either $K_{a,b}$, $(a-1)(b-1)=m$, or $K^c_{a;b}$, $2\leq c<a\leq b$ and $m+1=c+(a-1)(b-1)$. In addition, we establish some structural properties by the method of analysis to further reduce the candidate graphs.

Keywords: Spanning tree; Ferrers graph; Young diagram; Complete bipartite graph.

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